

3.6 – Solving Systems Using Matrices

Learning Targets for today

- ① To be able to represent a system of linear equations with a matrix.
- ① To be able to solve a linear system using the matrix.

Vocabulary

Matrix – A rectangular arrangement of numbers in rows and columns.

Ex: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $[x]$

Dimensions of a Matrix – Represents the number of rows and the number of columns in a matrix. (rows x columns)

Ex: $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (2 \times 2)$ $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = (2 \times 3)$

L \longrightarrow R

Element of a Matrix - The way to identify each element in a matrix.

Ex: a_{12} represents the element in row 1 column 2. $A = \begin{bmatrix} 12 & 32 \\ 44 & 55 \end{bmatrix}$ $a_{12} = 32$

Inverse Matrix – An $n \times n$ matrix with 1's on the main diagonal and 0's elsewhere.

Ex: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Ex: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$a(i) = a$

Coefficient Matrix – The matrix that represents the coefficients of the linear system.

Ex: $\begin{cases} ax + by = e \\ cx + dy = f \end{cases} \rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Matrix of Constants – The matrix that represents the constants of the linear system.

Ex: $\begin{cases} ax + by = e \\ cx + dy = f \end{cases} \rightarrow \begin{bmatrix} e \\ f \end{bmatrix}$

$$\left[\begin{array}{cc|c} a & b & e \\ c & d & f \end{array} \right]$$

Identifying a Matrix Element

Example for you...

Find a_{22} .

1. $A = \begin{bmatrix} 3 & 2 \\ 4 & 4 \end{bmatrix}$

$a_{22} = 4$

Your turn to try...

Find a_{21} .

1. $A = \begin{bmatrix} 6 & 4 \\ 4 & 3 \end{bmatrix}$

$a_{21} = 4$

Representing Systems with Matrices

Example for you...

Write the following system of equations with a matrix.

$$1. \begin{cases} 2x + 5y = 10 \\ 3x - y = 7 \end{cases} \quad \left[\begin{array}{cc|c} 2 & 5 & 10 \\ 3 & -1 & 7 \end{array} \right]$$

$$2. \begin{cases} x + 8y + 3z = 11 \\ 9x - 7y - z = 10 \\ y + 2z = 9 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 8 & 3 & 11 \\ 9 & -7 & -1 & 10 \\ 0 & 1 & 2 & 9 \end{array} \right]$$

Your turn to try...

Write the following system of equations with a matrix.

$$1. \begin{cases} 8x - 7y = 15 \\ 3x = 6 \end{cases} \quad \left[\begin{array}{cc|c} 8 & -7 & 15 \\ 3 & 0 & 6 \end{array} \right]$$

$$2. \begin{cases} x + 3z = 15 \\ 4x - 7y - z = 2 \\ x + y + 2z = 9 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 15 \\ 4 & -7 & -1 & 2 \\ 1 & 1 & 2 & 9 \end{array} \right]$$

Writing a Linear System from a Matrix

Example for you...

Write the following system of equations with a matrix.

1. $\left[\begin{array}{cc|c} 2 & 4 & 1 \\ -6 & 10 & 2 \end{array} \right]$ $\begin{cases} 2x+4y=1 \\ -6x+10y=2 \end{cases}$

Your turn to try...

Write the following system of equations with a matrix.

1. $\left[\begin{array}{cc|c} 8 & 4 & 6 \\ 4 & 0 & 2 \end{array} \right]$ $\begin{cases} 8x+4y=6 \\ 4x=2 \end{cases}$

Writing a Linear System from a Matrix

Example for you...

Write the following system of equations with a matrix.

1. $\left[\begin{array}{cc|c} 2 & 4 & 1 \\ -6 & 10 & 2 \end{array} \right]$

Your turn to try...

Write the following system of equations with a matrix.

1. $\left[\begin{array}{cc|c} 8 & 4 & 6 \\ 4 & 0 & 2 \end{array} \right]$

Solving a System using a Matrix

Example for you...

Solve following system of equations with a matrix.

1. ~~$$\begin{cases} 2x + 5y = 5 \\ -x + 2y = 7 \end{cases}$$~~

Solving a System using a Matrix

Example for you...

Solve following system of equations with a matrix.

1.
$$\begin{cases} 4x + 3y = 6 \\ -x - y = -1 \end{cases}$$

~~100%~~
$$\left[\begin{array}{cc|c} 1 & 0 & x=? \\ 0 & 1 & y=? \end{array} \right]$$

$$\left[\begin{array}{cc|c} 4 & 3 & 6 \\ -1 & -1 & -1 \end{array} \right]$$

$$\begin{array}{ccc} 4 & 3 & 6 \\ \hline 4(-1 & -1 & -1) \\ \hline 0 & -1 & 2 \end{array}$$

$$\left[\begin{array}{cc|c} 4 & 3 & 6 \\ 0 & -1 & 2 \end{array} \right]$$

$$\rightarrow 1(0 \ -1 \ 2)$$

$$\left[\begin{array}{cc|c} 4 & 3 & 6 \\ 0 & 1 & -2 \end{array} \right]$$

$$\begin{array}{ccc} 4 & 3 & 6 \\ \hline -3(0 & 1 & -2) \\ \hline 4 & 0 & 12 \end{array}$$

$$\left[\begin{array}{cc|c} 4 & 0 & 12 \\ 0 & 1 & -2 \end{array} \right]$$

$$\rightarrow 4(4 \ 0 \ 12)$$

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -2 \end{array} \right]$$

$$\rightarrow \frac{1}{4}(4 \ 0 \ 12)$$

$$\boxed{\begin{array}{l} x=3 \\ y=-2 \end{array}}$$